Color Harmonies and Color Spaces Used by Olivier Messiaen in *Couleurs de la cité céleste*

Paul Dworak  
College of Music  
Division of Music History, Theory, and Ethnomusicology

Abstract

This research explores the characteristics of the color spaces used by Oliver Messiaen in his orchestral composition *Couleurs de la cité céleste*. Messiaen discussed with Claude Samuel his experience of synesthesia, which is the ability of some persons to perceive colors when they hear sounds. In this work Messiaen identifies in the score the highly evocative colors and brilliance characteristics of gemstones suggested by scenes in the *Book of Revelation*, and he associates them with the voicing and instrumentation of the chords played by selected instruments from the orchestra that he uses in this work. Messiaen’s colors exist in a color space that corresponds with the formant space of the sound of the chords that realize these colors. Just as color models define hue, saturation and luminance in three dimensions, the $F_1 \times F_3 \times F_4$ formant space of Messiaen’s chords also locates the same color attributes within the three dimensions defined by these formants. The software application Speech Filing System determines the frequencies of the formants of the digitized sound of the orchestral chords that are associated with the various colors that he specifies. These formant data are plotted in three dimensions with DataFit, using nonlinear polynomial regression. These plots identify the surfaces and their orientations, as well as the distributions of formant frequency data and their trajectories, that correspond with colors and color combinations within this space. These data confirm that Messiaen’s colors are not merely symbolic associations, but represent his simultaneous perception of sound and color.

1. Introduction

Messiaen is somewhat unique among composers with synesthesia because he not only acknowledged possessing the ability but also used it explicitly to control some aspects of the harmonies and instrumental textures in several of his compositions. *Couleurs de la cité céleste* is Messiaen’s setting of the *Book of Revelation*. The author of this book describes the new holy city of Jerusalem (Rev 21:18-21):

The wall was constructed of jasper, while the city was pure gold, clear as glass. The foundations of the city wall were decorated with every precious stone; the first course of stones was jasper, the second sapphire, the third chalcedony, the fourth emerald, the fifth sardonyx, the sixth carnelian, the seventh chrysolite, the eighth beryl, the ninth topaz, the tenth chrysoprase, the eleventh hyacinth, and the twelfth amethyst. The twelve gates were twelve pearls, each of the gates made from a single pearl; and the street of the city was of pure gold, transparent as glass.
Messiaen sets these highly evocative colors and textures to chords played by selected instruments from the orchestra he uses in this work. His association between color and sound is not merely symbolic, but rather his notation of the sounds that he experiences with imagining the scene from Revelation.

Claude Debussy, an earlier French composer, exhibited the uncanny ability to hear the formant structure of the vowels sung by a vocalist, and to create a chord setting on the piano that had the same formant structure within its spectrum. Messiaen, likewise, captures his visual imagination in the sound structures that he includes in the score of his work.

1.1. The Nature of Synesthesia

Synesthesia is the capacity of some persons to be able to experience a phenomenon with more than one sense. In the case of music, a listener will both hear the music and perceive patterns of colors (photisms) that correspond with the sounds heard. Messiaen is, presumably, one of those synesthetes who can experience a bi-directional sensory association. In other words, he was able to hear the sound of colors and notate the orchestral chords that corresponded with them (Messiaen, 1993).

Some synesthetes perceive photisms as spatially extended. They often describe their perception as a set of patterns localized somewhere in space in front of them, with patterns that may move from left to right as the sound changes. Whether the patterns are perceived as external to the body or an internal phenomenon is irrelevant, however, because the perception is real. It also is involuntary, memorable, different for each synesthete, and persistent. Throughout a synesthete’s life, his or her color association with a particular sound will not change (Roberston and Sagiv, 2009).

Synethetes report a variety of photisms: thin lines, thick lines, parallel lines, curves, circles, spirals, etc. More than one photism may occupy the visual scene. Scenes with multiple colors may occur in various parts of the visual scene and move or dance among the photisms, or different colors may dissolve into a white connection (Campien, 2008).

Messiaen specifies not only colors and color combinations, but also their gemstone brilliance characteristics (e.g., topaze jaune, chrysoprase vert clair, et cristal). In this composition, Messiaen sometimes specifies a single color mixture (bleu violet) and at other times a mixture of two or more colors (rouge, orangé, et or). The current study does not speculate about the characteristics of the photisms that Messiaen perceives in mixtures of multiple colors.

1.2. Formant Analysis

This study is based on the assumption that synesthetes who associate colors with sounds will base their association on their formant analysis of the sound rather than a spectral analysis. A person’s ability to understand the meaning of a natural language is a skill that is more universal than the ability to comprehend the structure and harmonic organization of musical sounds.
understand language, a listener must be able to recognize vowels. A large literature describes how listeners identify vowels in any particular language based on their position in a two-dimensional formant space. When a speaker produces a vowel, his/her vocal mechanism (sinus, mouth, throat, etc.) amplifies certain harmonics of the spectrum produced by the speaker’s vocal chords. These bands of resonance are unique to individual vowels within any language, and the term “formant perception” describes a listener’s perception of the resonance bands produced by a speaker. Except in extreme registers, vowel perception is generally independent of the fundamental frequency produced by a speaker for any vowel, but the formant structure of vowels produced by men and women differ to a certain degree, as do the structure of vowels in different languages.

2. Related Research

2.1. Research into synesthesia

About thirty years ago, two of many students reported having synesthesia, which they said assisted them with melodic dictation in Aural Skills theory classes. Both, of course reported seeing different shapes and colors, and both had difficulty contextually discriminating between chromatic tones like D-flat and C-sharp, because they sound the same. The spelling of these notes is not heard but determined from context.

Almost twenty-eight years passed before two other students reported having synesthesia. Both of them reported that it made them an object of ridicule by their peers. This suggests that many students possess the ability, but few may report it.

One of the early studies of synesthesia was conducted by Guy Whipple with two subjects, and published in 1900. The results of his research are not much different from studies conducted with the last twenty years. Whipple reports:

“...The two given tones of the Appunn tonometer were followed by the appearance in the visual field of the closed eyes of light lines arranged horizontally against a luminous background, at times vaguely tinged with color, usually with pink or green. The higher tone generally appeared as a horizontal line above the lower; at times the line of the lower tone was darker. . . . The colors are not obtained by a deliberate attempt to see them; indeed, such an attempt rather tends to prevent their appearance. In general they are luminous, diaphanous clouds of color (though at times in definite figures) floating over a dusky background; and they are always seen projected to a position about 20 cm. from the eyes. They rarely cover the whole field. ”

Whipple provides numerous examples of the descriptions of photisms that were provided by the two subjects who listened to many sounds, ranging from a tuning fork to various classical

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compositions for orchestra. Carol Bergfeld Mills, Edith Howell Botelerô, Glenda K Larcombe conducted similar research, and their subjects drew the photism that they perceived, rather than reporting them verbally.²

A recent comprehensive study is that of Richard Cytowic, who discusses, among other topics, the experience of synesthetes, their developmental experience, medical reasons for synesthesia, neural circuitry involved in the experience, and the impact of synesthesia of a person’s personality and artistry.³

Cretien van Campen raises an interesting hypothesis that a baby’s brain processes and interprets sensory inputs as a single experience. They cannot separate sound, sight, and touch, for example. As they older and more experienced in associating certain sensory inputs with certain experiences and their meanings, the brain no longer needs information from all the senses, and specialization occurs in the neural circuitry. This theory suggests that adults with synesthesia never lost the capabilities that babies may possibly possess.⁴

2.2. Formant analysis and vowel spaces

Research into that type of synesthesia that involves sight and sound mostly focuses on the colors associated with musical sounds, or the sounds of vowels. By extension, studies have been made of the colors a synesthete may associate with the printed letters of words, or with numbers. Consequently, studies of how the brain processes vowel sounds is relevant to the present study because they suggest mathematical models that describe the nature of vowel sounds. Software applications that apply these models to digital sound files are now readily available, to make such research possible. As a matter of fact, the University College London Division of Psychology and Language Sciences has made Speech Filing System, an application for research into speech analysis and synthesis, available without cost, to encourage speech research.

The present study requires tools that visualize sound through its formant structure, and to compare sounds with one another in three-dimensional formant space. Understanding the relationships between sound and colors also require one or more metrics that relate to each other those sounds that Messiaen reports as being associated with colors in his composition. The following authors have conducted research related to the present study.

Klein, Plomp, and Pols provide an early study on the classification of vowels based on their formant frequencies.⁵ Alexandros Potamianosa) and Petros Maragos later discuss formant

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Berger similarly plots his nasality parameters against F1 and F2 to demonstrate how various parameters make possible speaker recognition based on the speaker’s nasality and language.

![Figure 2.1. Berger’s plot of nasality parameter A1-H1 against the F1 x F2 formant space. © 2007 by Michael Berger.](image)

Jansen determines the principal components of the representation of vowels by manifolds, based on the frequencies of a vowel’s spectrum that contribute most to recognition of the vowel. He then plots vowel data against three principal components axes as shown in Figure 2.2.

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2.3. The brain’s processing of color for perception.

Bartels and Zeki have studied the brain’s processing of color information and report. They that the brain brings together the “consciousnesses” that the brain creates in each level of visual processing into a single conscious experience.

“The human brain consists of many different visual areas. V1 and V2 have cells representing all the principal submodalities of vision (colour, form, motion and depth), cells concerned with a given submodality being segregated into anatomically identifiable compartments within both areas. . . . When we speak of the specialization of V4 for colour we do not mean to imply it is not also involved in the processing of luminance and of forms, both of which can be intimately linked to colour. . . . Activity at each stage or node of a processing perceptual system has a conscious correlate. Binding cellular activity at different nodes is therefore not a process preceding or even facilitating

More recently, James Fulton is conducting ongoing research into the brain’s processing of color vision, and he is a proponent of tetrachromatic vision in humans, with the wavelengths processed by the various neural channels shown in Figure 2.3.

![Neural model of human processing of color wavelengths](image)

Fulton defines a three-dimensional color space that explains the interpretation of the processing by the neural levels in the previous figure. In Figure 2.4, the red-green, and the green-blue dimensions are orthogonal, but there is also a blue-lilac dimension that is orthogonal to both in three-dimensions. Understanding the figure requires making it into a box, by cutting the top-left and bottom-left corners and folding them up to form the third dimension. The visible wavelengths of light are distributed as shown.
2.4. Cross modal perception

Coen states, \(^{11}\)

Coen states “We have thereby transformed our question about distance between regions into a question of similarity between their conditional spatial probability distributions in a co-occurring modality. This is computed via their Similarity distance.”

“In discussion of probability metrics, the notion of similarity generally follows directly from the definition of distance. Two distributions are deemed similar if the distributions between them are small according to some metric; conversely, they are deemed dissimilar when the metric determines they are far apart. In our approach, we will reverse this dependency. We first intuitively describe our notion of similarity and then formulate a metric that computes it in a well-defined way. We call this metric the Similarity Distance. . . . Our approach is applicable to comparing distributions over any metric space and has a number of interesting properties, such as scale invariance. . . .\(^{12}\)”


3. Method

The previous section explored some of the background of synesthesia, how it has been studied, and research into methodologies models of sound and vision. This section discusses in detail the steps followed in deriving and analyzing the data used in this study. First an outline of the method is presented:

1. Each chord in the score that was identified by Messiaen as being associated with a color was identified. In some cases, more than one successive chord was identified with the same color. Some colors occurred in various parts of the score.
2. From a recording of the composition, a sound editor was used to clip out the chords and place them in files based on their location. The chords associated with colors occurred at Rehearsals 11, 13, 24, 26, 73, 74, 75, and 76. (Rehearsal numbers are sequential numbers that a composer often writes above selected measures in a score, so that a conductor will have convenient and logical locations in the score to begin, when rehearsing the orchestra and studying the piece several measures at a time.)
   a. Special treatment was necessary for topaze jaune, chrysoprase vert clair, et crystal, because Messiaen notates in the score that the three clarinets are not part of the sounds that create the specified colors. Because it is impossible to delete the sounds of only selected instruments from a recording, these chords were synthesized with the score notation application Sibelius, and the clarinets were omitted in the synthesized chords associated with these colors.
3. Speech Filing System (SFS) is an application that analyzes digital sound files to determine their formant structure. It creates data files in text format that can be read by Microsoft Excel. Because of the dynamic nature of formant analysis, formants identified by SFS sometimes collided, and sometimes it was clear visually that the data value identified by SFS as the 3rd formant was really the 4th. These errors were corrected manually in Excel, but Excel was not used for any other processing.
4. Two applications were used for visual analysis of the data, DataFit 9.0, and IDL 7.0. IDL will be discussed later. Data for the the first, third, and fourth formants (F1, F3, and F4) were copied and pasted into DataFit. This application then used polynomial regression to identify the surfaces that approximated the distribution of formant frequency values in the three dimensional space F1 x F3 x F4. The application also created various two dimensional plots (F1 x F3, F1 x F4, etc.) and identified the curve, and parameters of its formula, determined by quadratic regression.
5. The various plots were compared to determine the distribution of formant frequency data points that identified each color.
6. In many cases, Messiaen associated two or three colors with a chord or series of chords. For the synesthete, this is perfectly logical, because the photisms that they see usually consist of mixtures of colors, which may be fields of individual colors that blend into each other, or they may be a mosaic of interspersed colors that shimmer and move about on the photism perceived by the synesthete. In order words, for Messiaen, the mixed colors were a single experience, and therefore required a single chord. However, comparing the distribution of formant frequency data points for different color
combinations that shared a single color made it possible to determine the characteristic location of that color in three dimensional formant space.

The above identification of the components was a brief summary. Now each will be discussed in greater detail.

3.1. Identification of Chords in the Score

As noted above, the chords the Messiaen associates with colors are located only at certain rehearsal numbers in the score. Figure 3.1a and b shows the chords at Rehearsal 13. Note that the color association begins where its name is written in the score, and continues until the next color identification is written, or when the Rehearsal section ends. The sections that include such chords are limited to the use of chordal texture only. All the voices of the chords change at the same time. The highly contrapuntal texture used elsewhere in the work is never used in sections associated with colors.

Figure 3.1a. Chords Associated with Colors, Rehearsal 13.
This figure shows another two characteristics of the color associations. The first two colors are associated with gems, and what are referred to as their brilliance characteristics. The first set are the three gems *topaze jaune, chrysoprase vert clair, et crystal*. The second is the pair *émeraude verte, améthyste violette*. Many gems have facets that are based on their crystalline structure, and the various facets reflect different shades of a hue, based on lighting and the position of the observer. One might also imagine walking around the celestial city and looking at the courses of gems that make up the foundation, and see their colors shimmer while walking, because of the brilliance characteristics of the gems.

The third set of colors is *rouge, orangé, et or*. This is a set of multiple colors, but they are not associated with gems. Of course, one could assert that *or* is not only the color gold, but also the metal. Associating *orangé* with the fruit, however, is not logical in the context of the scene from *Revelation*. *Rouge* and *orangé* are most likely abstract colors that interact in Messiaen’s photism. At rehearsal 75, Messiaen indicates the color combination *orangé, or, blanc laiteux*. In this case, he uses the designation “milky white” as a type of hue, and not as an indication of a substance.

Messiaen identifies single colors at Rehearsal 75 (*bleu violet*) and at 76 (*violet*). *Bleu violet*, in
this context, is a color mixture, although it is difficult to determine if he perceived blue and violet in his photism, or the single hue blue-violet.

Appendix A provides a Sibelius score with a summary of the notes of the chords and their orchestration, but it is not true to the rhythm nor the duration of the chords in the original score, because this score is for reference only.

3.2. Editing the Recording

The recording used as the digital source for the sounds of *Couleurs de la cité céleste* was *Messiaen*, by the Netherlands Wind Ensemble, under Conductor Reinhert de Leeuw, (Chandos Recording, 1994). The sound editor GoldWave created clips of the sounds at the rehearsal numbers noted above, and exported them as .wav files sampled at 11kHz. Individual chords from these sound files were identified by Speech Filing System, and these were saved as individual files (one for each chord), which facilitated analysis of the sounds.

As noted above, recording *topaze jaune, chrysoprase vert clair, et crystal* and eliminating the clarinets from the sounds was impossible. The sound of the Sibelius notation for the chord (Appendix A) was used instead to create the necessary sound files for this research.

3.3 Formant Analysis

Speech Filing System is an application dedicated to the analysis and synthesis of speech. It consists of a series of functions that calculate spectra, fundamental pitch lines, formant structure, etc. Figure 3.2 shows a cross-section of the second chord from Rehearsal 24 associated with the color *sardoine rouge*. This figure shows the waveform, the dynamic spectrum, the formant filter response, averaged over the duration of the sound, and the average frequency of the first six formants.
To calculate the dynamic evolution of the formants requires the following steps:

1. Resample the sound file to a 10kHz sampling rate, for the requirements Linear Predication analysis function (SFS function resamp).
2. Identify the pitch period Tx, as it evolves through time. (SFS function TX)
3. Run the Linear Prediction analysis function (SFS function HQanal), which generates a high resolution table of formant frequency values.
4. Run the Formants Estimation function (SFS function fmanal).
5. Display the results of HQanal and fmanal to determine which provides the most stable and detailed data.
6. Export the data to a text file.
7. Open the text file in Microsoft Excel to identify and correct collisions of formants and other errors that are visually obvious.

This script is summarized in Figure 3.3.
Figure 3.3. SFS Formant Analysis script.

Figure 3.4 shows the original waveform, the high-quality formant frequency estimation, and the lower resolution formant frequency estimation for Bleu Violet_2_75, the second chord at Rehearsal 75 that Messiaen associates with bleu violet.
Figure 3.4. Formant frequency estimation by SFS functions HQanal and fmanal.

SFS can export the frequency data from the formants estimation track to a text file that can be imported by Microsoft Excel. Figure 3.5 shows an example of the output of SFS function HQanal for Bleu Violet_2_75.

Figure 3.5. Table of first five formants for the second chord associated with bleu violet at Rehearsal 75.
The formant output for this chord includes the time point, as well as the frequency, bandwidth, and amplitude for each formant for each time. This research only used the formant frequencies. Bandwidth has been associated in the literature with vowel recognition (de Cheveigné, 1999). Lee, Kim, and Kang discuss the importance of bandwidth in speaker recognition. In a similar fashion, Fant and Mártony (1963) discuss the influence of the amplitude of the first formant on vowel recognition.

Figure 3.6a and b displays the distribution of formant frequency data points for this chord. In the first diagram, the plot of the lower resolution formant calculation is drawn on top of the plot of the high resolution calculation. In the second diagram, the position of the plots is reversed. Comparing the plots shows that they contain the same data, but the high resolution plot has more data, and a wider spread of the frequency data in the F1 x F3 data plane. Depending on the researcher’s needs, either could be better. The higher resolution plot tends to show better than the low resolution plot how data points on the fringe of the regression analysis surface relate to the entire cloud of data points.

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Figure 3.6. Comparing the output of the low resolution formant frequency analysis `fmanal` with that of the high resolution function `hqanal`.

### 3.4. Plotting the Formant Frequency Data

Once Microsoft Excel has opened the text file of the high- or low-resolution data, obvious errors of missing formant frequency data (a frequency of 0), or frequencies that clearly are in the wrong column (collision of formants), can be corrected. Figure 3.7 shows corrections highlighted in yellow. This screen excerpt has more corrections than is typical. The *bleu violet* color has greater variability in the frequency of its formants over time, which will result in collisions of its formants. Examples of this are easily seen in Figure 3.4.
Figure 3.7. Corrections to the formant data for Bleu Violet_2_75.

Once these corrections have been made, importing the data into DataFit is done by copying and pasting the data that the researcher wishes to view. Figure 3.8. shows how the data for these data looks on input.
Figure 3.8. Formant frequency data for Bleu Violet_2_75 imported into DataFit.

The researcher determine which column the data should be in. In this example, a three-dimensional surface will be calculated by regression analysis. The ride side of Figure 3.8 shows the nine eleven-parameter models that have already been run by the application. There are a total of 255 regression models that can be run.

In this example, Y is F4, X1 is F3, and X2 is F1. This will display F4 as a function of F3 and F1 in three dimensions. The regression surface will be drawn relative to, and rising from, the F1 x F3 plane. F1 and F3 can also be assigned to Y is the researcher wishes to see how these formants change as a function of the other formant frequencies.

Because the objective of the regression analysis is to find the parameters of a curve or a surface that explain or predict the Y data points, DataFit recalculates Y (in this case F4) and determines the resultant error. Figure 9 shows an excerpt from the regression output.
Figure 3.9. Recalculation of Y by the eleven parameter regression model displayed at the top of the figure.

Plotting these data and the surface calculated by this regression model shows us the three-dimensional surface that is reproduced in Figure 3.10.
Figure 3.10. The three-dimensional plot of formant frequency data and regression surface for Bleu Violet_2_75.

DataFit allows this diagram to be rotated, so that various perspectives can be viewed. Figure 6 showed an example of a rotation that showed the data in the F3 x F1 plane.

3.5. Examination of the data

The steps described above were followed for each chord associated with each color by Messiaen. Appendix B shows the waveform cross section for each sound (plot of its waveform, formant shape and formant frequencies. It also shows the spectrum and the low-resolution formant estimation for each sound. Appendix C shows the formant frequency samples for each color that are imported into DataFit.

Appendix D is derived from the application IDL. This is also a graphics application that displays curves and surfaces as does DataFit. IDL’s graphics are less clear to view, but it has superior calculation capabilities compared with DataFit. Appendix D shows the output of iTools, a function of IDL that calculates the distributions of formant frequency data points, displays this information in text, and calculates various histogram plots of the distributions. Figure 3.11 shows an example of these data and histograms for Bleu Violet_2_75.
Figure 3.11. iTools data for F1, F3, and F4 of Bleu Violet_2_75.
The mean frequency of the formants, their variance, and their skewness define the distribution of the frequency samples in the three-dimensional formant space. These data in turn determine a trajectory in each pair of dimensions (F1 x F3, F1 x F4, F3 x F4) for these samples within the space. The trajectory in each dimension, which is identified by polynomial regression, is one of the most unique features of each color, and a trait that differentiates colors from one another.

Appendix E contains all of the color trajectories computed to date. Still to be completed are the F1 x F4 for trajectories many of the colors. The color planes and curves for Bleu Violet_2_75 include:

Figure 3.12. Surface and curve trajectories for formant frequency samples in three- and two-dimensions.
Figure 3.13 presents Figure 3.12d in greater detail.

As noted above, two-dimensional trajectories were all modeled exclusively with the formula

$$Y = a \cdot X^2 + b \cdot X + c$$

In the figure, $a = -2.95E-03$, $b = 3.35$, and $c = 942.77$. Because $a$ is negative, the direction of curvature is down. When results of the data analysis are discussed below, they show that the curvature and length of the trajectories is a way in which colors are located in three-dimensional formant space, and by which colors relate to one another in this space.

Appendix F contains a complete set of histograms for the formants, those for F4 as well as F1, F3, and F4 displayed together. The location of the formants and the shape of their skewness value help to explain relationships among colors. For example violet and the color pair yellow/green share a common trajectory in F3 x F1 space.
Figure 3.14. Trajectories of violet and yellow/green in F3 x F1 space.

Both have the same direction of curvature and nearly the same degree of curvature, based on their values of a. Both have similar rotation with respect to the Y (F3) axis, based on their values of b, and if the Violet curve were continued, it would be slightly higher than the Yellow/Green curve, based on their values of c. In James Fulton’s color model, discussed below, these colors are nearly on the same axis in this color wavelength space, and here display a similar characteristic.

Violet \[ F3 = -1.67 \times 10^{-3} \times F1^2 + 3.35 \times F1 + 532.07 \]
Yellow/Green \[ F3 = -2.29 \times 10^{-3} \times F1^2 + 4.93 \times F1 + 423.26 \]

Figure 3.15. Relationship of Violet and Green/Yellow in James Fulton’s color space, © 2001 James Fulton.
Appendix G contains all of the regression data for the two-dimensional curve-fitting performed by DataFit. An example for Bleu Violet_2_75 is shown. These regression data determine the values for the parameters a, b, and, c, as well as whether the curve has a high probability of representing the formant frequency distribution.

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</table>

Figure 3.16. An example of regression data for Bleu Violet_2_75.

3.6. Summary

This section has presented the entire methodology for generating formant frequency data from a digital recording of the chords that Messiaen associated with colors and plotting it in a variety of ways, to determine the characteristic features and locations of the colors in the three-dimensional formant space. The next section reviews findings to date.

4. Results

A number of models of color perception were studied to see if any corresponded to the distribution data for Messiaen’s colors in three-dimensional formant space. Figure 4.1 shows James Fulton’s Luminance/Chrominance Diagram for Research, the axes and scales of which are based on data describing human perception of color. This model does not explain all of the data describing the formant structure of the sounds Messiaen’s uses, but it does show relationships among colors that correspond somewhat with the data from formant analysis.
Figures 4.2, 4.3, and 4.4 compare the color trajectories of red, blue, and green in three-dimensional formant space. Figure 4.2 shows the three-dimensional formant frequency data distribution. Figure 4.3 rotates Figure 3 to provide a view of the F1 x F3 plane (from above). Figure 4.4 plots F1 x F4 in two-dimensions and shows the quadratic curves that fit these distributions along both axes.

The formant frequency data for *sardoine rouge* are distributed narrowly in F3 and F4, but more widely along F1, resulting in these data describing a trajectory parallel to the F1 axis. None of Messiaen’s chords represent pure blue or pure green. However, *rouge, taché de bleu* can be expected to include features of both colors. The data points are more widely distributed in three-dimensional formant space, but they continue to run parallel to the F1 axis, expected for red, and also to have a strong vertical distribution, which data for other chords in the work confirms are representative of blue. The data for *émeraude verte, améthyste violette* are widely distributed and their trajectories run diagonal to both axes. Once again, other chords that include the color green have data distributed from low values of F1 and F4 to higher values respectively, which creates the diagonal trajectories for these distributions.

Figures 4.5, 4.6, and 4.7 show distributions of formant frequencies and their trajectories for
bleu violet; orangé, or, et blanc laiteux; and émeraude verte, bleu saphir, et or. Figure 4.1 shows that the blue and yellow hues are separated by the greens. The trajectories for violet (the darker purple color in the figures) run parallel to both the F1 and F4 axes, with about equal distribution, as would be expected of a mixture of red and blue. The orange and gold colors show a similar distribution and trajectories, but the curvature of these respective trajectories are opposite from those of violet. The green-blue-gold formant frequencies (the aqua color) fall in a narrow range, and their vertical distribution (dark blue curve) has the same direction of curvature, and nearly the same location in formant space, as violet. The horizontal distribution (the lowest orange curve) is parallel to, and shares the same direction of curvature as the orange and gold colors, which suggests that these colors have a functional relationship to each other.

Figure 4.2: Three-dimensional formant frequency data for sardoine rouge; rouge taché de bleu; émeraude verte, améthyste violette.
Figure 4.3: Two-dimensional formant frequency data (F3 x F1) for sardoine rouge; rouge taché de bleu; émeraude verte, améthyste violette.

Figure 4.4: Two-dimensional formant frequency data (F1 x F4) for sardoine rouge; rouge taché de bleu; émeraude verte, améthyste violette.
Figure 4.5: Three-dimensional formant frequency data for bleu violet; orangé, or, blanc laiteux; émeraude verte, bleu saphir, et or.

Figure 4.6: Two-dimensional formant frequency data (F3 x F4) for bleu violet; orangé, or, blanc laiteux; émeraude verte, bleu saphir, et or.
Figure 4.7: Two-dimensional formant frequency data (F1 x F4) for bleu violet; orangé, or, blanc laiteux; émeraude verte, bleu saphir, et or.

These data demonstrate that Messiaen’s chords and orchestration correspond to colors through the frequency relationships of their formant frequencies and the distribution of these frequencies in F1 x F3 x F4 formant space. Regression analysis clearly shows that reds have a larger range of F1 frequencies than many other colors, but that their F3 and F4 distributions are very small. Blues, violets, and color combinations that contain them have low F1, F3, and F4 frequencies. Greens have the largest distributions, and contain the highest F3 and F4 frequencies of all colors. Orange and yellow colors parallel reds to a degree but have larger F3 and F4 frequency distributions than red shades.

Clearly, there is no linear relationship among the colors and color distributions. Consider that Messiaen is not imagining the colors as lines or circles, but rather as patterns of colors in two-dimensional space (as we might experience in looking into a kaleidoscope). The fact that even the positive or negative curvature of colors, as well as their frequency distributions, show consistent relationships between groups of colors and their components (as in Figure 4.7) indicates that a linear model is not appropriate.

Insofar as pitch recognition is based on a logarithmic scale, the data shown above was plotted in logarithmic space, but those plots did not reveal any new relationships. Instead, colors like red showed a more focused trajectory in formant space, but most other colors overlapped in
the space such that they were not distinguishable. Insofar as color perception is not based on a logarithmic frequency model, there is no reason to assume that a logarithmic pitch space would map to a linear color space, but more study into this correspondence is planned for the future.

5. Pitch Class Set/Color Correspondence

Much music of the twentieth century lends itself to pitch-class (PC) set analysis. This analytical technique focuses on the interval structure of chords rather than primarily on their harmonic function, as would be the case with analysis in tonal theory. The components of PC set analysis are:

- The use of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, and E to correspond respectively with the notes C, C#/Db, D, D#/Eb, E, F, F#/Gb, G, G#/Ab, A, A#/Bb, and B (T and E represent 10 and 11).
- The correspondence between numbers and note names is a modulo-12 naming system, equivalent to the way that numbers are arranged on a clock that represents Universal Time, with the hour after 1000 being 1100, and the hour after 1100 being 0000 (when am and pm are used).
- Composers combine pitch classes into sets of notes that form the vertical and horizontal arrangement of notes in the score. Typically, sets with three to nine members are studied.
- Intervals between pitch classes are derived by subtracting pitch class numbers. The interval between pc 0 and pc 3 is 3, which corresponds with a minor 3\(^{rd}\) or augmented 2\(^{nd}\) in traditional nomenclature.
- Sets form interval class (IC) vectors, which always have six digits. Each digit corresponds with an interval size, including half-step (IC 1), whole-step (IC 2), minor 3\(^{rd}\)/augmented 2\(^{nd}\) (IC 3), major third (IC 4), perfect fourth (IC 5), and tritone (IC 6). Each digit represents the count of intervals of that size that are present in the set. Intervals larger than a tritone are included with the count of the intervals that are their inversion. The perfect 5\(^{th}\), IC 7, is included in the count of IC 5, the perfect 4\(^{th}\), and the major 7\(^{th}\), IC 11, is included in the count of IC 1, etc.\(^\text{16}\)

Messiaen acknowledged using “modes of limited transposition” in his works, which consist of simple sets that are grouped into larger structures.

The final component of this research project has been to study the correlation between PC sets, the chords that Messiaen notates in his score, and the color that he specifies (also studied by Bernard, 1986).\(^\text{17}\) Two cases are straightforward. The orchestration for bleu violet uses various transpositions of the single 4-note set (tetrachord) 4-z29, which consists of pitch classes 0137, and the IC vector [111111]. This set is referred to as the “all-interval set” because its pitch

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classes generate one instance of each interval class from the half-step to the tritone. The set which corresponds with violet is the tetrachord 4-9, 0167, with IC vector [220002]. (See Appendix A.)

To determine whether colors correspond specifically with the interval contents of pitch class sets, the following score was generated by the application Sibelius, and the audio files of each chord were analyzed by SFS. Figure 5.1 shows the two arrangements of the notes of each set.

Figure 5.1. Two arrangements of tetrachords 4-z29 (0137) and 4-9 (0167).

Figure 5.2 shows the formant frequency correspondence between bleu violet_2_75 and 0137 in both close and open position. Figure 5.3 shows the correspondence between violet_3_76 and 0167 in both close and open position. In general, the first, and either the third or fourth formant are very close in center frequency. It is somewhat surprising that either position of the set would correspond with the color, which suggests that the IC vector and not the octave distribution of the notes is important to the correspondence. The fact that the instrumentation and voicing of the sets is critical to the composition suggests that the harmonics generated by the notes of the sets, and the interaction among these harmonics, is essential to generating the formant structure of the various sounds. More research on this topic is needed.
Comparing Figures 5.2 and 5.3 also shows that bleu violet and violet share many formants frequencies, because they are shades of one color, and exist in adjacent locations of the color plane.

Determining the correspondences between other colors or color combinations and multiple sets that constitute the voicing of instruments is a difficult problem that will continue to be researched in the future.

6. Conclusions

The component pitch classes, interval spacing, and orchestration of each of the chords that Messiaen associates with a color generate a formant structure that defines a surface in F1 x F3 x F4 formant space. The total body of data studied show that formant frequency data for reds are centered in the F3 and F4 dimensions and run parallel to F1. Blues have a formant frequency distribution that begins below those of red in the F4 dimension, and increases in both the F3 and F4 dimensions, while varying little in F1. Green includes high formant frequencies in F1, F3 and F4, consistently higher than those for red, and the distribution of these frequencies is the greatest of any color. The remaining colors (violet, yellow, orange, etc.) cluster about red, but not in a fashion that corresponds with Fulton’s model in Figure 2.

The following conclusions might be drawn:

1. The three-dimensional formant frequency space that represents the colors that Messiaen specifies is nonlinear, being stretched along the F1 x F3 x F4 diagonal as the values on these axes increase linearly, as displayed in Figures 4.2 to 4.7. Future work will explore compensating for this nonlinearity. Fulton acknowledges that his model...
currently does not represent a similar characteristic of the human perception of colors.

2. The trajectories of the formant frequency distributions describe curves derived from nonlinear regression performed on the data. These trajectories, particularly in F1 x F4 space, best differentiate colors and show the relationships among them.

3. Mixtures of colors share trajectories with the component colors, if these components occur separately in other chords in the work and can be analyzed separately.

4. The data also suggest that narrow distributions of formant frequencies represent saturated colors, and wider distributions represent colors that are more diffuse.

5. The current data does not disclose the photisms that Messiaen perceived. If this were possible, it would explain how colors related spatially in his virtual visual field, which would explain distributions of formant frequency data.

Clearly, however, Messiaen’s ability to associate sounds with colors in a way that maps some characteristics of formant space with those of color space demonstrates the unity of his perception of sound and color, and the consistency of his sensory associations.

7. Publications

To date, parts of this research were published in the Proceedings of the Eleventh International Conference on Music Perception and delivered as a poster presentation at the conference in Seattle, WA, August 23-27, 2010.
Bibliography


1. Applicant: Paul E. Dworak
2. Emplid: 10001984
3. Phone #: 940-565-4906
4. E-mail Address: dworak@unt.edu
5. Department/Division: Music History, Theory, and Ethnomusicology
6. College/School: College of Music
7. Full-Time (Yes or No): Yes
8. Current Rank: Professor
9. Years at UNT: 30
10. Year Tenured: 1985
11. Proposal Title: Color Harmonies and Color Spaces Used by Olivier Messiaen in *Couleurs de la cité céleste*

12. Abstract (Provide an abstract (200 words maximum) of the proposed project, which can be understood by a person NOT familiar with your discipline.):

This research project explores and attempts to define the color spaces used by Oliver Messiaen to generate the color harmonies that he uses in his orchestral composition *Couleurs de la cité céleste*. Messiaen experienced synesthesia, which is the ability of some persons to perceive colors when they hear sounds, or in general to experience a sensory input with more than one of the human senses. Messiaen explicitly defined in the score those colors that correspond with the harmonies and instrumental textures in this composition. *Couleurs de la cité céleste* is Messiaen’s setting of the *Book of Revelation*, which is replete with references to color and texture when describing the new city of Jerusalem. This research uses software analysis of a digital recording of the composition to identify the three-dimensional formant space that describes Messiaen’s color harmonies. Humans use formant analysis to identify vowel sounds in human speech, and synesthetes often report perceiving colors that correspond with vowel sounds as well as with musical sounds. This research project is unique because previous music theory research has focused on the intervals in the chords that Messiaen uses, and the correlations postulated do not work in all cases. Work completed to date on this project show clear orientations in two- and three-dimensional formant spaces for the various color combinations used by Messiaen in this composition.
13. Describe the specific outcomes that will result from this project (1-page maximum):

If this proposal is accepted and funded, the faculty salary will enable me to spend one month completing this research during a time when I am not teaching. The outcomes for this research are to:

1. Complete a formant analysis on all the chords that Messiaen associates with colors in *Couleurs de la cité céleste*
2. Identify the orientations of colors and mixed colors in linear three-dimensional formant space
3. Define a color space that normalizes the orientations of the colors in three-dimensional formant space
4. Determine the principles that Messiaen is used in his selection of color harmonies in this composition
6. Begin the preparation of papers that will be submitted to journals, including the following: *Music Perception, Music Theory Spectrum, Music and Mathematics.*

14. Project Narrative (Provide a narrative description (3-page maximum, 11-point type) of the project you intend to carry out, and how the RCE Award would facilitate the project’s completion.): (Attached)
RCE AWARD APPLICATION —BUDGET SUMMARY
Grant Period: January 1 through December 31 (with expenditures completed by August 31 if possible)

Budgetary Restrictions:
- Summer salary requests may not exceed 1-month (dollar-value based upon current 9-month salary)
- Travel expenses may not exceed $2000
- Other expenses (in any category below) may not exceed a total of $1000

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